Algebraic Integrability of Discrete and Continuous Dynamical Systems Defined by Birational Plane Mappings.

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Discrete and continuous dynamical systems defined by arbitrary birational maps F_n of degree $n \geq 2$ from the Cremona group $\mathrm{Cr}CP_k^N$ with coefficients from algebraic closed field k are considered. Here $N \equiv 2$, $k \equiv \mathbb{C}$. The existence conditions for algebraic invariants in extended phase space and algorithm of their construction for such systems are presented. The main idea of our approach to this problem is based on the new concept — the decomposition of the sets $\mathbf{O}$, $\mathbf{O}^{-1}$ of indeterminacy points of F_n, F_n^{-1} relatively of the action of the corresponding inverse map F_n^{-1}, F_n. This decomposition ($\mathbf{O} = \mathbf{O}^{(int)} \bigoplus \mathbf{O}^{(rest)}$ and $\mathbf{O}^{-1} = \mathbf{O}^{-1^{(int)}} \bigoplus \mathbf{O}^{-1^{(rest)}}$) establishes a one-to-one correspondence between pairs of equivalent points in $\mathbf{O}^{(int)}$ and $\mathbf{O}^{-1^{(int)}}$ which are the end points of orbits. Dynamics of birational maps is obtained.